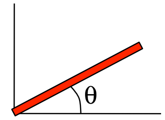
Problem 10.3

door as viewed from above, swinging about origin



A door's angular position (symbol "θ") is defined by:

$$\theta = 5.00 + 10.0t + 2.00t^2$$

a.) For t = 0, determine:

angular position:

$$\theta = 5.00 + \cancelto{0}{10.0t} + \cancelto{0}{2.00t^2}$$
$$= 5.00 \text{ radians}$$

angular speed:

$$\omega = \frac{d\theta}{dt}$$
$$= \frac{d\left(5.00 + 10.0t + 2.00t^2\right)}{dt}$$
$$= 10.0 + \cancelto{0}{4.00t}$$
$$= 10.0 \text{ rad/sec}$$

Knowing that $\theta = 5.00 + 10.0t + 2.00t^2$

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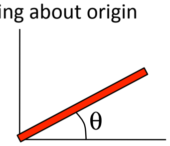

the angular acceleration (symbol "α")?

There are two ways to do this. If all you have is "$\theta(t)$," you can use:

$$\alpha = \frac{d^2\theta}{dt^2}$$

Note that this is the rotational equivalent of knowing "x(t)" and writing:

$$a = \frac{d^2x}{dt^2}$$

Or if you know "$\omega(t)$," which you do, you can write:

$$\alpha = \frac{d\omega}{dt}$$
$$= \frac{d(10.0 + 4.00t)}{dt}$$
$$= 4.00 \text{ rad/sec}^2$$

With $\theta = 5.00 + 10.0t + 2.00t^2$:

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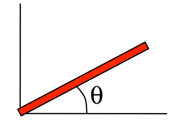

b.) For t = 3.00 seconds, determine:

angular position:

$$\theta = 5.00 + 10.0t + 2.00t^2$$
$$= 5.00 + 10.0(3.00 \text{ s}) + 2.00(3.00 \text{ s})^2$$
$$= 53.0 \text{ radians}$$

angular speed:

$$\omega = \frac{d\theta}{dt}$$
$$= \frac{d\left(5.00 + 10.0t + 2.00t^2\right)}{dt}$$
$$= 10.0 + 4.00(3.00 \text{ s})$$
$$= 22.0 \text{ rad/sec}$$

angular acceleration:

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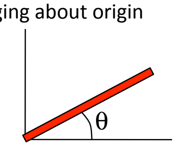

$$\alpha = \frac{d\omega}{dt}$$
$$= \frac{d(10.0 + 4.00t)}{dt}$$
$$= 4.00 \text{ rad/sec}^2$$

You are now done with the problem proper. What follows is a HUGE NOTE you really should read.

Forces, velocities, accelerations, these are all VECTORS. That is, they have a magnitude, a "line" and a direction along that line. Hence, you know that writing out $\vec{v} = -(3 \text{ m/s})\hat{i}$ means the object is moving with velocity magnitude of 3 m/s, it's traveling along the *x-axis* (that's what the "i-hat" denotes) and its moving in the *negative direction* along that line. You know, in short, how to decode this information to learn something about the body's motion.

It shouldn't be surprising to find the the rotational counterparts of force and velocity and accelerations are also VECTORS and can be presented in a unit vector notation. So what does $\vec{\omega} = -(3 \text{ rad/s})\hat{i}$ tell you in a rotational setting? Well:

The magnitude part is obvious. It is tell you how many radians the body is rotating through per second.

You need to identify the plane in which the rotation is taking place. We do this by identifying *the AXIS about which the rotation takes place* (that is the only direction that isn't changing during the motion as the body's velocity vector is constantly *changing direction*). Sooo, the unit vector defines the direction of the *axis or rotation* in the knowledge that that direction if *perpendicular* to the *plane of the motion.* (For our example, according to this, the axis is along the *x-direction*, which means the rotation is in the *y-z plane*.

And lastly, the only other thing that has to be accounted for is whether the rotation is clockwise or counterclockwise (as viewed from the *positive* side of the axis about which the rotation is taking place). That is what the sign does with a *positive sign* denoting *counterclockwise* rotation and a *negative sign* denoting a *clockwise rotation.*

With all of this in mind, you really don't have to worry about inserting unit vectors into the problems you will be doing for the AP test. Why? Because a "one dimensional" problem is one associated with rotation in *one plane only*. As the unit vector used to denote the axis about which the rotational occurs will be the same for ALL the variables involved, there is no need to include it. You will have to include *positive* and/or *negative* signs because it does matter whether a body has angular velocity that is oriented clockwise or counterclockwise, but that is really the only nod you have to make toward the vector nature of these problems.

In short, if you have a body rotating in the *x-z plane* with some initial angular position and some initial positive angular velocity and some negative angular acceleration, a kinematic equation you could write out, technically would be:

$$|\theta_2|(\hat{i}) = |\theta_1|(\hat{i}) + |\omega_1|(+\hat{i})(\Delta t) + \frac{1}{2}|\alpha|(-\hat{i})(\Delta t)^2$$

Fortunately for you, to use this equation all you'd need write is:

$$\theta_2 = \theta_1 + \omega_1(\Delta t) + \frac{1}{2}(-\alpha)(\Delta t)^2$$